2550-04

Fall 2024

Test 1 Solutions

(1)

$c_1=1, c_2=1, c_3=0: \; 1\cdot\langle 1,0\rangle + 1\cdot\langle 1,1\rangle + 0\cdot\langle 2,1\rangle = \langle 2,1\rangle$

$c_1=1, c_2=-1, c_3=1: \; 1\cdot\langle 1,0\rangle - 1\cdot\langle 1,1\rangle + 1\cdot\langle 2,1\rangle = \langle 2,0\rangle$

$c_1=0, c_2=0, c_3=0: \; 0\cdot\langle 1,0\rangle + 0\cdot\langle 1,1\rangle + 0\cdot\langle 2,1\rangle = \langle 0,0\rangle$

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(2)

(a) $\vec{a} - 2\vec{b} = \langle 3,1,0\rangle - 2\langle 1,-1,2\rangle$
$= \langle 3,1,0\rangle + \langle -2,2,-4\rangle = \boxed{\langle 1,3,-4\rangle}$

(b)
$\|\vec{c}\| = \sqrt{0^2+1^2+2^2+(-3)^2+4^2} = \sqrt{1+4+9+16} = \boxed{\sqrt{30}}$

(c) $\vec{a}\cdot\vec{b} = \langle 3,1,0\rangle\cdot\langle 1,-1,2\rangle = (3)(1)+(1)(-1)+(0)(2)$
$= 3-1+0 = \boxed{2}$

$\vec{c}\cdot\vec{d} = \langle 0,1,2,-3,4\rangle\cdot\langle 1,0,-1,0,1\rangle$
$= (0)(1)+(1)(0)+(2)(-1)+(-3)(0)+(4)(1)$
$= -2+4 = \boxed{2}$

3 (a)

$$2B+F = 2\begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} 3 & -3 \\ 3 & 1 \end{pmatrix}}$$

(b)

$$BA = \begin{pmatrix} 2 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} (2\ -1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (2\ -1)\begin{pmatrix} 1 \\ 1 \end{pmatrix} & (2\ -1)\begin{pmatrix} -1 \\ 0 \end{pmatrix} \\ (1\ 1)\begin{pmatrix} 1 \\ 2 \end{pmatrix} & (1\ 1)\begin{pmatrix} 1 \\ 1 \end{pmatrix} & (1\ 1)\begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 2-2 & 2-1 & -2+0 \\ 1+2 & 1+1 & -1+0 \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 1 & -2 \\ 3 & 2 & -1 \end{pmatrix}}$$

(c) $$AC = \underbrace{\begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \end{pmatrix}}_{2\times 3}\underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{2\times 1}$$

$3 \neq 2$

$\boxed{\text{not possible}}$

(d) $$D^T = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}^T = \begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

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$$\left(\begin{array}{ccc|c} 2 & 1 & -7 & 6 \\ 1 & 2 & -2 & 0 \\ 1 & -4 & 1 & -6 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 2 & 1 & -7 & 6 \\ 1 & -4 & 1 & -6 \end{array}\right)$$

$$\xrightarrow[-R_1+R_3 \to R_3]{-2R_1+R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 0 & -3 & -3 & 6 \\ 0 & -6 & 3 & -6 \end{array}\right)$$

$$\xrightarrow{-\frac{1}{3}R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 0 & 1 & 1 & -2 \\ 0 & -6 & 3 & -6 \end{array}\right)$$

$$\xrightarrow{6R_2+R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 9 & -18 \end{array}\right)$$

$$\xrightarrow{\frac{1}{9}R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & 2 & -2 & 0 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -2 \end{array}\right)$$

$$\begin{aligned} x + 2y - 2z &= 0 \\ y + z &= -2 \\ z &= -2 \end{aligned}$$

leading variables: x, y, z
free variables: none

$$\begin{aligned} &(1)\quad x = -2y + 2z \\ &(2)\quad y = -2 - z \\ &(3)\quad z = -2 \end{aligned}$$

$$\begin{aligned} &(3)\quad z = -2 \\ &(2)\quad y = -2 - z = -2 + 2 = 0 \\ &(1)\quad x = -2y + 2z = 0 - 4 = -4 \end{aligned}$$

Answer

$x = -4$
$y = 0$
$z = -2$
one solution

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$$\boxed{\begin{aligned} x + y - 2z + w &= 1 \\ z - 3w &= 0 \\ w &= -1 \end{aligned}}$$

Already reduced
leading: x, z, w
free: y

$$\boxed{\begin{aligned} x &= 1 - y + 2z - w & (1)\\ z &= 3w & (2)\\ w &= -1 & (3)\\ y &= t & (4) \end{aligned}}$$

(4) $y = t$

(3) $w = -1$

(2) $z = 3(-1) = -3$

(1) $x = 1 - y + 2z - w = 1 - t + 2(-3) - (-1)$
$= 1 - t - 6 + 1$
$= -4 - t$

Answer:

$x = -4 - t$
$y = t$
$z = -3$
$w = -1$

6 HW 1 - Part 2

Problem 2(e)